Math 2D Quiz 1 Morning - January 14, 2016

Please put name and ID on *both* sides for grading and redistribution.

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[Based on 10.1.[redacted]].

Show all of your work.

*There is a question on the back side.

1. Let $x = \sqrt{t}$, $y = 1 - t$ parameterize a curve, where $0 < t < \infty$.

3 pts (a) Eliminate the parameter to find a Cartesian equation of the curve and sketch the curve. Indicate, with arrows, the direction in which the curve is traced as the parameter increases. (Hint/Be Careful Graphing: If $0 < t < \infty$, what are the ranges of x and y values?)

2 pts (b) Compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. Leave your final answers as functions of t.

a) There are 2 ways to eliminate the parameter.

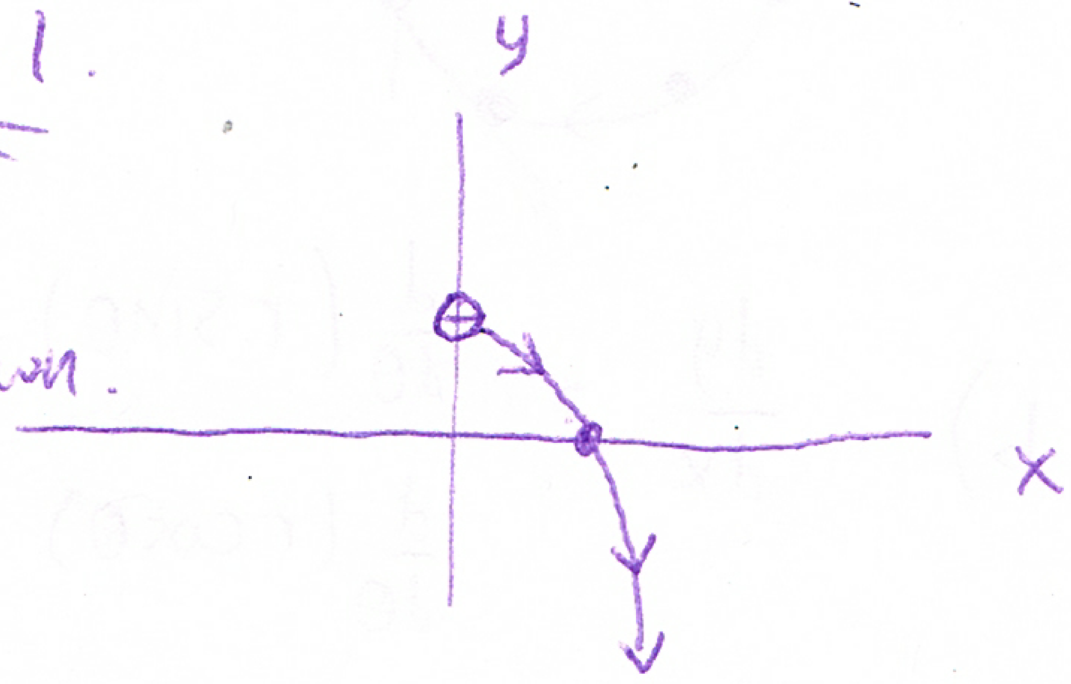
(i) $x = \sqrt{t} \Rightarrow t = x^2 \Rightarrow \boxed{y = 1 - x^2}$ (ii) $t = 1 - y \Rightarrow \boxed{x = \sqrt{1-y}}$

Hint / Careful Graphing: $0 < t < \infty$ implies $x > 0$, $y < 1$.

So, we only get right half of $y = 1 - x^2$ curve,

At $t \approx 0$, we are at $(0,1)$ so as t increases, we go down.

$t = 1$: we are at $(1,0)$



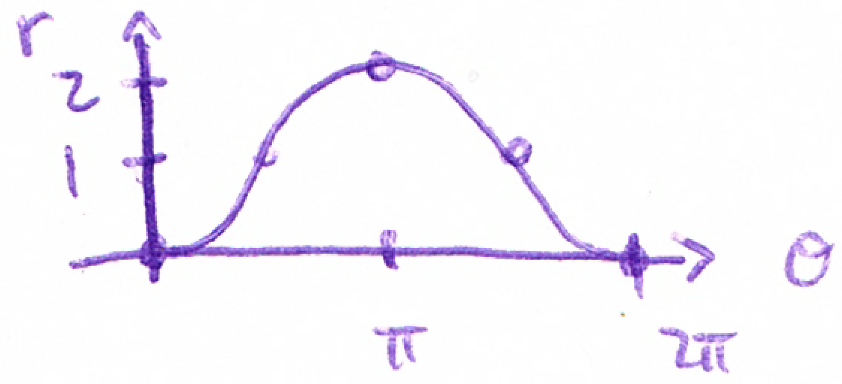
b) $\bullet$ $\frac{dy}{dx} \equiv \frac{dy/dt}{dx/dt} = \frac{-1}{\frac{1}{2\sqrt{t}}} = \boxed{-2\sqrt{t}}$

[Note: This equals $-2x$ since $x = \sqrt{t}$, as we would expect with $y = 1 - x^2$].

$\bullet$ $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{-2 \cdot \cancel{\frac{1}{2\sqrt{t}}}}{\cancel{\frac{1}{2\sqrt{t}}}} = \boxed{-2}$

[Also expected from $y = 1 - x^2$].

★ Some people graphed $r = 1 - \cos\theta$ like "$y = 1 - \cos x$"



instead of tabulating pts

[Based on 10.3.63]

2. Let $r = 1 - \cos\theta$.

2 pts (a) Sketch the curve.

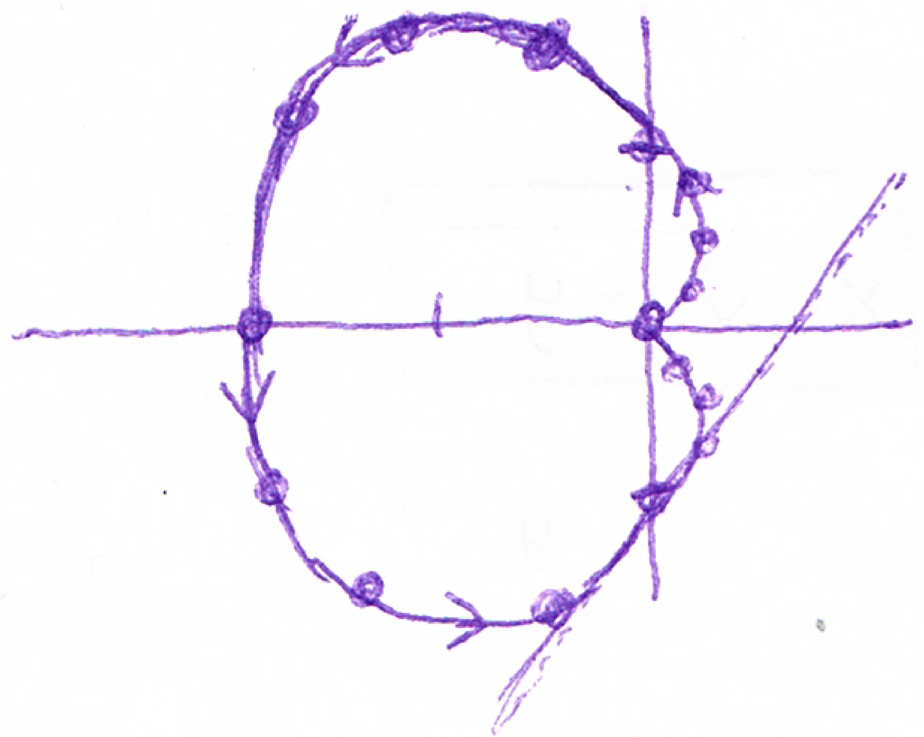
2 pts (b) Compute $\frac{dy}{dx}$ as a function of θ.

(c) When $\theta = 3\pi/2$: What is the (x, y) coordinate on the curve? What is the value of $\frac{dy}{dx}$?

1 pt (d) Using part (c), write the equation of the tangent line to the curve when $\theta = 3\pi/2$.

a)

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$\frac{5\pi}{6}$	π	$7\pi/6$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$
r	0	$1-\frac{\sqrt{3}}{2}$	$1-\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	1	$3/2$	$1+\frac{\sqrt{2}}{2}$	$1+\frac{\sqrt{3}}{2}$	2	$1+\frac{\sqrt{3}}{2}$	$1+\frac{\sqrt{2}}{2}$	$\frac{3}{2}$	1	$\frac{1}{2}$	$1-\frac{\sqrt{2}}{2}$	$1-\frac{\sqrt{3}}{2}$



part (d) tangent line.

★ A shortcut is to note: $\cos\theta$ is <u>even</u>.

★ <u>Recall</u>: $x = r\cos\theta$

$y = r\sin\theta$ // plug in $r = 1 - \cos\theta$

b) $$\frac{dy}{dx} = \frac{\frac{d}{d\theta}(r\sin\theta)}{\frac{d}{d\theta}(r\cos\theta)} = \frac{\frac{d}{d\theta}(\sin\theta(1-\cos\theta))}{\frac{d}{d\theta}(\cos\theta(1-\cos\theta))} = \frac{\frac{d}{d\theta}(\sin\theta - \sin\theta\cos\theta)}{\frac{d}{d\theta}(\cos\theta - \cos^2\theta)}$$

$$= \boxed{\frac{\cos\theta - \cos^2\theta + \sin^2\theta}{-\sin\theta + 2\cos\theta\sin\theta}}.$$

This also equals $\frac{\cos\theta - \cos 2\theta}{-\sin\theta + \sin 2\theta}$.

c) At $\theta = \frac{3\pi}{2}$:

- $x = r\cos\theta = 0$
- $y = r\sin\theta = 1 \cdot \sin\frac{3\pi}{2} = -1$

$\rightarrow \boxed{(x, y) = (0, -1)}$

- $$\left.\frac{dy}{dx}\right|_{\frac{3\pi}{2}} = \frac{\underbrace{\cos 3\pi/2}_{\text{zero}} - \cancel{\cos^2\frac{3\pi}{2}} + \sin^2\frac{3\pi}{2}}{\underbrace{-\sin\frac{3\pi}{2}}_{} + 2\cancel{\cos\frac{3\pi}{2}}\sin\frac{3\pi}{2}} = \frac{1}{-(-1)} = \boxed{1}$$

$\frac{3\pi}{2} = 0$

d) Using (c), it is <u>$y + 1 = x$</u> <u>$(y = x - 1)$</u>. It looks appropriate on the graph too.